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**INFLUENCE OF THE BAUSCHINGER EFFECT  
ON RESIDUAL STRESS AND FATIGUE LIFETIMES  
IN AUTOFRETTAGED THICK-WALLED CYLINDERS**

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13. ABSTRACT (Maximum 200 words) This work addresses the influence of Bauschinger effect upon residual stresses and associated fatigue lifetimes for pressurized, autofrettaged thick cylinders. The model employed allows for the variation with radius of Bauschinger Effect Factor (BEF) throughout the autofrettaged tube since the percentage plastic strain, which determines BEF, will vary from a maximum value at the bore to zero at the elastic-plastic interface.  Accounting for BEF variability, it is demonstrated that the residual compressive hoop stress at the inner radius of the tube reaches a maximum value at the percentage overstrain level below which reversed yielding does not occur. Existing experimental residual stress measurements from a variety of sources are shown to support this thesis. This value of overstrain may serve to maximize crack initiation lifetime in autofrettaged thick cylinders.  For a tube with significant heat-checking and associated initial crack-like defects, it is necessary to consider fatigue crack growth rates governed by a crack growth law such as Paris's Law. For a tube of radius ratio 2.0 and at a value of approximately 40% overstrain, slightly in excess of that for the onset of reversed yielding, the fatigue lifetime exhibits a maximum value. Fatigue lifetimes achieve a maximum value at overstrain levels in which yielding reaches 1.4 times bore radius and are almost constant thereafter. Furthermore, such extended overstrain leads to a small increase in residual stress at the outside diameter (OD), thus increasing R ratio at that location and reducing fatigue lifetime for crack growth originating at the OD. Existing experimental lifetime measurements are shown to require the inclusion of BEF to properly account for these observed lifetimes.					
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## TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION .....	1
ANALYSIS PROCEDURE .....	3
INFLUENCE OF BAUSCHINGER EFFECT ON RESIDUAL STRESSES .....	8
INFLUENCE OF BAUSCHINGER EFFECT ON FATIGUE LIFETIME .....	10
EXPERIMENTAL LIFETIME DATA .....	10
A NOTE ON YIELD STRENGTH MODIFICATION .....	12
SUMMARY AND CONCLUSIONS .....	12
FOOTNOTE .....	13
ACKNOWLEDGMENTS .....	13
REFERENCES .....	14

## TABLES

1. Summary of Hydraulic Fatigue Tests of 188 mm Inner Diameter  
A723 Steel Thick Walled Cylinders with 0 and 50% Overstrain ..... 11

## LIST OF ILLUSTRATIONS

1. Bauschinger Effect: (a) Stress-Strain Curve for Typical Gun Steel;  
(b) Variation of BEF with Percentage Plastic Strain; (c) Form of Residual  
Hoop Stress Predictions; (d) Form of Residual Hoop Stress Measurements . 16
2. Tube Geometry ..... 17
3. Plastic Strain, Bauschinger Effect Factor, and Residual Hoop Stress  
as a Function of Radius ..... 18
4. Comparison of Predicted Stresses with Experimental Data ..... 19
5. Plastic Strain, Bauschinger Effect Factor, and Residual Hoop Stress  
at the Bore ..... 20

6.	Residual Stress at the Bore - Bauschinger Model and Experimental Evidence .....	21
7.	Predicted Lifetimes as a Function of Percentage Overstrain .....	22
8.	Effect of Overstrain and BEF on Measured Fatigue Lifetime of Pressurized Cylinders .....	23

## INTRODUCTION

The use of autofrettage to enhance fatigue lifetimes of thick cylinders subjected to internal cyclic pressurization is well known and relatively well understood. The determination of residual stresses arising from the autofrettage process is important for fatigue life estimation [1]. The residual stress and strain distribution in a thick cylinder which has been pressurized until yielding spreads outwards from the bore to a known radius and which then has the internal pressure removed has been analyzed by Hill [2]. Hill's analysis is based upon ideal elastic-perfectly plastic assumptions and standard yield criteria, normally those due to Tresca or Von Mises, with identical magnitude of yield strength in tension and compression.

For many years workers have acknowledged the probable influence of the Bauschinger effect [3] which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. This phenomenon is illustrated in Fig. 1(a) wherein the yield strength in tension is  $Y$  and the yield strength in compression is  $-fY$ ; the data in Fig. 1(a) are based upon work by Clark [4].  $f$  is sometimes termed the *Bauschinger Effect Factor* (BEF); work by Milligan et al [5] provides a relationship between tensile plastic overstrain and the BEF; the latter varies from unity at zero plastic strain, drops rapidly with increasing plastic strain and saturates at around 2% plastic strain, being effectively constant thereafter. This saturation value of BEF is designated  $f^*$ . The variation of BEF, based upon [5], is illustrated in Fig. 1(b).

**Fig. 1 : Bauschinger Effect, Relevant Earlier Work:**  
(a) Stress-Strain Curve for Typical Gun Steel (after[4])  
(b) Variation of BEF with Percentage Plastic Strain (after[5])  
(c) Form of Residual hoop Stress Predictions (after[8,9])  
(d) Form of Residual Hoop Stress Measurements(after[6])

The reduction of compressive yield strength within the yielded zone of an autofrettaged tube is of importance because, on removal of the autofrettage pressure, the region near the bore experiences high values of compressive hoop stress, approaching the normal tensile yield strength of the material if the unloading is totally elastic. If the combination of hoop and radial stresses exceeds some yield criterion the tube will re-yield from the bore thus losing much of the potential benefits of autofrettage. There is convincing experimental evidence, based upon X-ray and other residual stress measurements, for this effect [6,7]. A cruder measure of this effect is obtained by radial slitting of tubes to release the locked-in bending moment created by the autofrettage stresses; any level of opening angle below the 'ideal' value predicted by the elastic-perfectly plastic analysis is an indication of Bauschinger effect [1].

Most prior work on the stress analysis of autofrettaged tubes which encompasses the Bauschinger effect has assumed a constant value of BEF throughout the autofrettaged tube [4,8]. This is clearly incorrect as a general assumption, since the percentage plastic strain will vary from a maximum value at the bore of the autofrettaged tube to zero at the elastic-plastic interface. The most striking feature of residual hoop stress predictions based upon such modelling is an abrupt change of slope at the reversed yielding interface with a subsequent decrease in the magnitude of compressive hoop stress towards the bore; the value at the bore being  $-\frac{1}{2}\sigma_y$ , as required by the yield criterion. This is illustrated in Fig. 1(c).

In one case Chen [9] has selected a constant value of  $f$  as a function of plastic strain at the bore (based upon data in [5]) and the continued 'compressive strain hardening' slope (also illustrated in Fig. 1(c)). Chen employed a constant value for the slope,  $m'$ , of 0.3. The most obvious impact of incorporating this quasi-strain hardening effect is that the abrupt change of slope at the elastic plastic reversed yielding interface is dramatically reduced and that the magnitude of compressive hoop stress continues to increase all the way to the bore. At this stage it is hypothesized that much of this predicted shift is caused by the selection of a model of unbounded strain hardening which does not match the experimental evidence for a given value of plastic strain; furthermore, by adopting a single constant value for  $m'$  there is an implicit assumption that the slope (even if it were an adequate proxy for the non-linear process) is constant for all values of prior tensile plastic strain.

Whilst there is a reasonable body of experimental work on measuring residual hoop stresses via X-ray [4,6], neutron-diffraction [7] and hardness/acoustoelasticity [10] very few have sufficient data points to investigate the critical, near-bore, reversed yielding zone arising from the Bauschinger effect. Only two such have been located, the most useful of which is due to Lee et al [6]; these results consistently show a significant change of slope at the reversed yield interface and a significant subsequent decrease in hoop residual compressive stress as the bore is approached, Fig. 1(d). The other work which contains a significant number of data points near the bore is due to Frankel et al [10]; these measurements relate to hardness but may be interpreted in terms of residual stress. Reference [10] consistently demonstrates the same form of slope change and reduced residual stress as [6].

The objectives of the work presented herein are to:

- a. Develop a simple analysis procedure to include variations in plastic strain and hence BEF through the wall thickness which conforms with experimentally observed residual stress distributions, particularly in the near-bore region.
- b. Conduct a parameter study to identify optimum values of percentage overstrain in order to maximize fatigue lifetimes of tubes having different diameter ratios.

c. Comment upon the implications for fatigue lifetime of increases in material yield strength.

## ANALYSIS PROCEDURE

References [3, 4, 8 and 9] provide all necessary analyses required to develop a model for the autofrettage process with a yield strength in tension of  $Y$  and a yield strength in compression of  $-fY$ . The equations required for this analysis are summarized below. Note that the various references have been assembled adopting a consistent set of definitions, and presenting only those details required for this particular analysis.

**Fig. 2 : Tube Geometry**

The following definitions apply (see Figure 2): Tube inner radius,  $a$ ; tube outer radius,  $b$ ; radius of plastic zone at peak of autofrettage cycle,  $c$ ; maximum radius of reversed plasticity,  $d$ ; general radius location,  $r$ :

Tube with internal pressure  $p$ , hoop ( $\sigma_\theta$ ) and radial ( $\sigma_r$ ) stresses are:

$$\sigma_\theta = \frac{a^2 p}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \quad (1)$$

$$\sigma_r = \frac{a^2 p}{b^2 - a^2} \left[ 1 - \frac{b^2}{r^2} \right] \quad (2)$$

The pressure,  $p^*$ , to cause yielding out to radius  $r = c$  is :

$$p^* = Y \ln(c/a) + \frac{Y}{2b^2} (b^2 - c^2) \quad (3)$$

Pressure for initial yielding at the bore,  $p_i^*$ :

$$p_i^* = \frac{Y}{2b^2} (b^2 - a^2) \quad (4)$$

Pressure  $p_y^*$  for complete yielding out to radius  $b$ :

$$p_y^* = Y \ln(b/a) \quad (5)$$

If tube is pressurized to  $p^*$  ( $p_i^* < p^* < p_y^*$ ) there is partial yielding to radius  $c$ , with associated stresses:

$$\sigma_\theta^* = -p^* + Y(1 + \ln(r/a)) \quad a \leq r \leq c \quad (6)$$

$$\sigma_r^* = -p^* + Y \ln(r/a) \quad a \leq r \leq c \quad (7)$$

$$\sigma_\theta^* = \frac{Yc^2}{2b^2} \left[ 1 + \frac{b^2}{r^2} \right] \quad c \leq r \leq b \quad (8)$$

$$\sigma_r^* = \frac{Yc^2}{2b^2} \left[ 1 - \frac{b^2}{r^2} \right] \quad c \leq r \leq b \quad (9)$$

If pressure  $p^*$  is subsequently completely removed, assuming yield stress in compression of  $-Y$  and that unloading is entirely elastic with no reversed yielding from the bore (valid provided  $b/a < 2.22$ ), the residual stress distribution is given by:

$$\sigma_\theta^R = -p^* + Y(1 + \ln(r/a)) - \frac{p^* a^2}{(b^2 - a^2)} \left[ 1 + \frac{b^2}{r^2} \right] \quad a \leq r \leq c \quad (10)$$

$$\sigma_r^R = -p^* + Y \ln(r/a) - \frac{p^* a^2}{(b^2 - a^2)} \left[ 1 - \frac{b^2}{r^2} \right] \quad a \leq r \leq c \quad (11)$$

$$\sigma_\theta^R = \left[ \frac{Yc^2}{2b^2} - \frac{p^* a^2}{(b^2 - a^2)} \right] \left[ 1 + \frac{b^2}{r^2} \right] \quad c \leq r \leq b \quad (12)$$

$$\sigma_r^R = \left[ \frac{Yc^2}{2b^2} - \frac{p^* a^2}{(b^2 - a^2)} \right] \left[ 1 - \frac{b^2}{r^2} \right] \quad c \leq r \leq b \quad (13)$$

Clearly, a re-pressurization to a pressure  $p < p^*$  will produce a stress distribution which may be calculated by the addition of (10) and (1), (11) and (2), (12) and (1), (13) and (2).

Modelling Bauschinger effect by assuming a reduced compressive yield strength of  $(-fY)$ , i.e. constant and independent of radius, there is the possibility of yielding from the bore out to radius  $d$  as the original autofrettage pressure,  $p^*$  is removed. In this region of reversed plasticity Tresca's criterion applies and the stresses are:

$$\sigma_{\theta} = -fY(1 + \ln(r/a)) \quad a \leq r \leq d \quad (14)$$

$$\sigma_r = -fY \ln(r/a) \quad a \leq r \leq d \quad (15)$$

which satisfies the two requirements that  $\sigma_r = 0$  at  $r = a$  and Tresca's criterion,  $\sigma_{\theta} - \sigma_r = -fY$  in the range  $a \leq r \leq d$ .

Consider now the elastic region  $r \geq d$ . As a result of removal of the autofrettage pressure,  $p^*$  and of the reversed yielding process the elastic-plastic interface at  $r = d$  experiences a radial stress  $\sigma_r (= -p^{REV})$  given by equation (15) minus equation (7), thus :

$$-p^{REV} = \sigma_r \Big|_{r=d} = p^* - (1 + f)Y \ln(r/a) \quad (16)$$

Thus the stresses in the region of elastic unloading are composed of (6) and (8), (7) and (9) plus some additional (negative) pressure  $p^{REV}$  applied at  $r = d$  as a result of unloading and reversed plasticity; this pressure produces stresses given by:

$$\sigma_{\theta} = \frac{d^2 p^{REV}}{b^2 - d^2} \left[ 1 + \frac{b^2}{r^2} \right] \quad d \leq r \leq b \quad (17)$$

$$\sigma_r = \frac{d^2 p^{REV}}{b^2 - d^2} \left[ 1 - \frac{b^2}{r^2} \right] \quad d \leq r \leq b \quad (18)$$

The requirement for the outer region  $d \leq r \leq c$  is that at  $r = d$  the material is just yielding. The total stresses  $\sigma_{\theta}^T$  and  $\sigma_r^T$  given by superposition of (6) and (17), (7) and (18) are:

$$\sigma_{\theta}^T = \frac{d^2 p^{REV}}{b^2 - d^2} \left[ 1 + \frac{b^2}{r^2} \right] - p^* + Y(1 + \ln(r/a)) \quad d \leq r \leq c \quad (19)$$

$$\sigma_r^T = \frac{d^2 p^{REV}}{b^2 - d^2} \left[ 1 - \frac{b^2}{r^2} \right] - p^* + Y \ln(r/a) \quad d \leq r \leq c \quad (20)$$

but Tresca's criterion applies at  $r = d$ , thus:

$$\sigma_\theta^T - \sigma_r^T = -fY \quad \text{at } r = d \quad (21)$$

and from (19), (20) and (21):

$$-fY = Y + p^{REV} \left[ \frac{2b^2}{b^2 - d^2} \right] \quad (22)$$

but the interface pressure is given by (16) which combines with (22) to give:

$$\frac{p^*}{Y} = (1 + f) \left( \left[ \frac{b^2 - d^2}{2b^2} \right] + \ln(d/a) \right) \quad (23)$$

Substituting from ((16) into (19) and (20), recognizing that radial stress and pressure are of opposite sign:

$$\sigma_\theta^T = -\{p^* - (1 + f)Y \ln(r/a)\} \frac{d^2}{b^2 - d^2} \left[ 1 + \frac{b^2}{r^2} \right] - p^* + Y(1 + \ln(r/a)) \quad d \leq r \leq c \quad (24)$$

$$\sigma_r^T = -\{p^* - (1 + f)Y \ln(r/a)\} \frac{d^2}{b^2 - d^2} \left[ 1 - \frac{b^2}{r^2} \right] - p^* + Y \ln(r/a) \quad d \leq r \leq c \quad (25)$$

Superimposing (8) and (17), (9) and (18) and substituting from (16):

$$\sigma_\theta^T = \left[ 1 + \frac{b^2}{r^2} \right] \left[ \frac{Yc^2}{2b^2} - \{p^* - (1 + f)Y \ln(r/a)\} \frac{d^2}{b^2 - d^2} \right] \quad c \leq r \leq b \quad (26)$$

$$\sigma_r^T = \left[ 1 - \frac{b^2}{r^2} \right] \left[ \frac{Yc^2}{2b^2} - \{p^* - (1+f)Y \ln(r/a)\} \frac{d^2}{b^2 - d^2} \right] \quad c \leq r \leq b \quad (27)$$

Equations (10) to (13) together with (3) define the residual stress field after removal of autofrettage pressure when there is no reversed yielding, whilst equations (14) and (15), (24) to (27) together with (3) and (23) define the residual stress field in instances where reversed yielding with a constant value of  $f$  occurs.

For reversed yielding not to occur on unloading:

$$(\sigma_\theta - \sigma_r)_{r=a} < -fY \quad (28)$$

thus, from (10) and (11):

$$p^* < \frac{(1+f)Y}{2} \left[ \frac{b^2 - a^2}{b^2} \right] \quad (29)$$

or in terms of the pressure for initial yielding  $p_i^*$ , eqn (4), for no reversed yielding:

$$p^* < (1+f)p_i^* \quad (30)$$

For example, for a cylinder  $b/a = 2$ ,  $f = 0.5$   $p^* < 1.5p_i^*$  then from (4) and (3):

$$1.5p_i^* = 0.5625Y = Y \ln(c/a) + \frac{Y}{2b^2}(b^2 - a^2) \quad (31)$$

and a straightforward iterative process gives  $c/a = 1.33$ ; thus any overstrain in excess of 33% will cause reversed yielding at the bore. Clearly, in general it will be necessary to solve equation (23) numerically in order to calculate  $d$ .

Tensile plastic hoop strains at the peak of the autofrettage cycle are reported in [9]. The variation of plastic strain (correcting for a typographical error in Eqn (8) of [9]) is:

$$\epsilon^p = \frac{2Y}{\sqrt{3}E} (1 - \nu^2) \left[ \frac{c^2}{r^2} - 1 \right] \quad a \leq r \leq c \quad (32)$$

where  $E$  is Young's Modulus and  $\nu$  is Poisson's ratio. In order to extend the above analysis to accommodate the possibility of the variation of BEF with radius, i.e.  $f = f(r)$  we note that equations (14) and (15) become:

$$\sigma_\theta = -f(r)Y(1 + \ln(r/a)) \quad a \leq r \leq d \quad (33)$$

$$\sigma_r = -f(r)Y \ln(r/a) \quad a \leq r \leq d \quad (34)$$

which satisfy the two requirements that  $\sigma_r = 0$  at  $r = a$  and Tresca's criterion,  $\sigma_\theta - \sigma_r = -f(r)Y$  in the range  $a \leq r \leq d$ .

## INFLUENCE OF BAUSCHINGER EFFECT ON RESIDUAL STRESSES

Milligan et al [5] have provided extensive experimental data for the material of interest, namely a modified AISI 4330 steel having a martensitic structure. Figure 1(b) shows their results for this steel, based upon 0.1% and 0.2% offset yield strength, however the procedure which follows may be generalised simply by curve-fitting equivalent data for other materials.

Milligan's data is conveniently fitted by:

$$\begin{aligned} f &= (1 - f^*) \left[ \frac{\tan(1 - \epsilon^p)}{\tan(1)} \right]^n + f^* & \dots \epsilon^p < 2\% \\ f &= f^* & \dots \epsilon^p > 2\% \end{aligned} \quad (35)$$

where  $f^* = 0.47$  and  $0.35$  represent the 'saturation' values at 2% plastic strain for 0.2% and 0.1% offsets respectively and  $n = 4.5$ .

Equation (32) provides a plot of plastic strain versus radius. A typical example, for the case  $a = 50$  mm,  $b = 100$  mm and 65% overstrain is shown in Fig. 3(a). Individual values of  $\epsilon^p$  may now be converted, via (35) to provide a graphical indication of  $f(r)$ . This is also shown on Fig. 3(a). Finally, by reference to (33) we can examine the variation of hoop residual stress with radius resulting from variable  $f(r)$ . Hoop stress values for a fixed value of  $f$ , namely  $f(a)$ , and for the variable form,  $f(r)$  are presented in Fig. 3(b). *Note that the variations shown are only valid within the zone of reversed plasticity ( $r < d$ ) and that an iterative procedure is required for the precise calculation of  $d$ .*

### **Figure 3 : Plastic Strain, Bauschinger Effect Factor and Residual Hoop Stress as a Function of Radius**

The earlier assertion that residual stresses based upon a constant value of  $f$  will not match those determined using  $f(r)$  is clearly validated. Furthermore the residual hoop stresses at the unloading interface are seen to become more compressive when  $r$  variation is incorporated.

For comparison with available experimental results obtained by X-ray diffraction residual hoop stress predictions for a tube  $a = 57$  mm,  $b = 152.4$  mm,  $Y = 1200$  MPa

with 74% overstrain, reference [6], are presented in Fig. 4. Considering the complexities of both the experiment and the model, agreement is considered good.

***Figure 4 : Comparison of Predicted Stresses with Experimental Data(from Reference [6])***

To fully develop an understanding of the impact of BEF varying with radial position,  $f(r)$  consider a tube of radius ratio ( $b/a$ ) of 2.0 with various percentage overstrains. Confining attention to the bore we calculate plastic strain, BEF and compressive hoop stress at the inner radius of the tube. Figure 5 shows the predictions of the model and leads to an interesting observation. The magnitude of compressive hoop residual stress at the bore increases and then decreases slightly with increasing percentage overstrain. The overstrain level below which reversed yielding does not occur is clearly associated with the radius for maximum compressive stress.

***Figure 5 : Plastic Strain, Bauschinger Effect Factor and Residual Hoop Stress at the Bore***

In order to validate this prediction Fig. 6 shows experimental results from a variety of available sources [4, 6, 7 and 10]. Fig. 6 is generalized and normalized by plotting hoop residual stress at the bore normalized with yield strength versus depth of autofrettage normalized with bore radius. Agreement is generally considered good for such a wide range of test and analysis conditions. Furthermore it is noted that the majority of the X-ray and neutron diffraction results lie very close to the prediction based upon 0.1% offset.

***Figure 6 : Residual Stress at Bore - Bauschinger Model and Experimental Evidence***

The logic of a presentation based upon  $c/a$  is straightforward. Equation (32) shows that plastic strain is effectively independent of outer radius, hence the BEF, being a function of plastic strain, is likewise independent of external radius. *(Note that there is actually a modest dependency on end conditions and external radius, [11], but this is ignored for the purposes of this simple model)*. This effect is somewhat surprising; the implication is that in those cases where Bauschinger effect occurs, residual stresses within the BAZ (Bauschinger affected zone) are identical for a given  $c/a$  ratio and are not dependent on outer radius. The Lamé stresses arising from any subsequent bore pressurization are, of course, a function of outer radius. Furthermore, removal of material from the OD after autofrettage serves to reduce bore residual hoop compressive stress (as a result of elastic unloading based upon a superposition of Lamé hoop stresses). Hence autofrettage should be conducted on the component as near to its final dimensions as is practicable in order to maximise bore hoop compressive stresses in the cases where reversed yielding may occur, generally  $c/a > 1.3$ , which encompasses virtually all autofrettaged gun tube geometries.

## INFLUENCE OF BAUSCHINGER EFFECT ON FATIGUE LIFETIME

Fatigue crack initiation may be governed by positive cyclic stress range at the initiation site. Hence the value of overstrain associated with maximum compressive hoop residual stress at the bore may serve to maximise initiation lifetime in autofrettaged thick cylinders.

However, in the case of interest here, namely a pre-fired gun tube with significant heat-checking and associated initial, crack-like defects it is necessary to consider fatigue crack growth rates governed by crack tip stress intensity factor range ( $\Delta K$ ) and a crack growth law such as Paris' Law [12]. Lifetimes will clearly be extremely sensitive to residual stresses at the bore, since this stress and the internal pressure dominate  $K$  for very short crack lengths at which most of the fatigue lifetime is expended. However, at longer crack lengths the depth and profile of residual stresses and the stresses due to internal cyclic pressurization become important, but have relatively less effect on lifetime

Fatigue lifetimes, based upon stress intensity factor solutions of extremely high accuracy (errors  $< 0.5\%$ ) determined by the Modified Mapping Collocation technique [13] and packaged as weight function data [14], are presented in Fig. 7. The calculations were based upon the following geometrical and materials properties:  $a = 50$  mm;  $b = 100$  mm; two initial, diametrically opposed bore cracks of length 0.5 mm; internal cyclic pressure 400 MPa; Young's modulus,  $E$ , 200 GPa; Yield Strength 1200 MPa; Paris Law coefficient,  $C$ ,  $6.52E-12$ ; Paris Law exponent,  $m = 3$ . The stress intensity factor calculations take full account of thru-the-thickness variation of residual and pressurization stresses. Overstrains from 0 to 100% were examined, and lifetimes calculated for the cases of ideal autofrettage,  $f(r) = 1$ , and incorporating Bauschinger effect,  $f(r)$  given by equation (35) for both 0.1% and 0.2% offsets.

### ***Figure 7 : Predicted Lifetimes as a Function of Percentage Overstrain***

At a value of approximately 40% overstrain, slightly in excess of that for the onset of reversed yielding, the fatigue lifetimes exhibit a maximum value. The conclusion is clearly that overstrains in excess of 40% for the wall ratio considered do not serve to increase fatigue lifetimes. Furthermore such an increase leads to a small increase in residual stress at the outside diameter (OD), thus increasing  $R$  ratio at that location and reducing fatigue lifetime for crack growth originating at the OD.

## EXPERIMENTAL LIFETIME DATA

Comparison of measured fatigue lifetimes from cannon tubes with predicted lifetimes from the analyses discussed here can be complicated by the prefiring and related heat-checking often present in cannon. Fortunately, some unfired cannon tubes were hydraulic fatigue tested to failure, including tubes with 0% and 50%

overstrain [15]. The tubes had a 188 mm inner diameter, 374 mm outer diameter, and rifling on the bore surface that produced a hoop stress concentration factor of 1.7 at the rifling fillet [16], where fatigue failure initiated. The material was forged ASTM A723 pressure vessel steel with nominal yield strength of 1060 MPa. Three tubes had no overstrain, and six had 50% overstrain. Since there was a clear difference in overstrain between two groups of tubes and no heat-checking complications, these results provide a useful experimental check on the analytical predictions described here. It will be interesting to compare the trend of overstrained tube lifetimes with those for no overstrain, on a stress range versus lifetime plot that considers the influence of Bauschinger effect on stress range. This comparison is considered next.

Table 1 lists conditions for the nine test results used in the comparison. The mean yield strength of the six overstrained tubes was lower than that with no overstrain, but the strengths were within the expected variation for these large forged tubes. The local stress range at the inner surface failure location was calculated from the following expression,

$$\Delta\sigma_{\theta} = p + k_T\sigma_{\theta} + f\sigma_{\theta}^R \quad (36)$$

where  $k_T$  is the stress concentration factor at the rifling fillet and the other terms

**Table 1: Summary of Hydraulic Fatigue Tests of 188 mm Inner Diameter A723 Steel Thick Walled Cylinders with 0 and 50% Overstrain**

overstrain %	yield strength MPa	stress range MPa	mean life cycles
0	1108	892	13,630
50%	1023	$f = 1.0$ : 375 $f = 0.5$ : 634	23,152

have been discussed. The local stress range includes: the effects of pressure,  $p$ , in the crack as it initiates and grows; the effects of  $k_T$  on the applied hoop stresses,  $\sigma_{\theta}$ ; and, most important for the current topic of concern, the influence of Bauschinger effect on the residual hoop stress due to overstrain, through application of the strength reduction factor,  $f$ , to the residual stress,  $\sigma_{\theta}^R$ . Note in Table 1 that stress range for 50% overstrain with no strength reduction, that is  $f = 1.0$ , is significantly lower than that with the Bauschinger effect included,  $f = 0.5$ . The important consequences of this difference in stress range can be seen in a plot of all the comparison results, Figure 8, discussed next.

**Figure 8 :Effect of Overstrain and BEF on Measured Fatigue Lifetime of Pressurized Cylinders**

The logarithmic plot in Figure 8 shows the stress range versus life results for the three cannon tubes with no overstrain and a straight trend line through these results with the -0.46 slope from a recent extensive comparison of cannon tube lives [16]. The results for the six cannon tubes with 50% overstrain are plotted in two ways, with and without the influence of Bauschinger effect. With no Bauschinger effect included in Equation 36, that is with  $f = 1.0$ , the local stress range is considerably lower, whereas with Bauschinger effect, with  $f = 0.5$ , the local stress range is higher. Note that for  $f = 0.5$  the overstrain results are well represented by the trend line, whereas for  $f = 1.0$  the overstrain results are considerably displaced from the trend line. Moreover, if the trend line were used to estimate a life for  $f = 1.0$ , the life estimate would be nonconservative, since it would be about twice that of the actual measured life. It is clear from these results that inclusion of the Bauschinger effect in the calculation of local stress range gives a much improved description of stress range and associated measured fatigue lifetime for overstrained cannon pressure vessels.

## A NOTE ON YIELD STRENGTH MODIFICATION

Calculations of fatigue lifetimes are frequently based upon Paris' law, referred to earlier. In performing such calculations it is clearly essential to take account of the loss of residual stress resulting from the Bauschinger effect. The significant effects are clearly demonstrated in the preceding sections.

Increasing the yield strength will offer some advantage by locking in higher residual stresses and thus reducing stress intensity range. Increased yield strength may also advantageously influence the Paris law exponent and/or coefficient. This process does, of course, raise the question as to whether the BEF may be a function of yield strength, with the possibility that any advantage may be wholly or partially lost as a result. Milligan et al [5] report 'no systematic variation of the Bauschinger effect (factor) with yield strength' for martensitic steels in the range of yield strengths from 735 MPa to 1120 MPa. However care should be taken in extrapolating this finding significantly beyond the upper yield strength level.

## SUMMARY AND CONCLUSIONS

This work addressed the influence of Bauschinger Effect upon residual stresses and associated fatigue lifetimes for pressurized, autofrettaged thick cylinders. It is demonstrated that it is necessary to allow for the variation with radius of plastic pre-strain and BEF throughout the autofrettaged tube since the percentage plastic strain, which determines BEF, will vary from a maximum value at the bore to zero at the elastic-plastic interface.

Accounting for BEF variability it is demonstrated that the residual compressive hoop stress at the inner radius of the tube reaches a maximum value at the overstrain level corresponding to the onset of reversed yielding. This value of overstrain may serve to maximise crack initiation lifetime in autofrettaged thick cylinders.

For a tube with significant heat-checking and associated initial, crack-like defects it is necessary to consider fatigue crack growth rates governed by a crack growth law such as Paris' Law. For a tube of radius ratio 2.0 and at a value of approximately 40% overstrain, slightly in excess of that for the onset of reversed yielding, the fatigue lifetime exhibits a maximum value. Overstrains in which yielding extends beyond 1.4 times bore radius do not serve to increase fatigue lifetimes. Furthermore such an increase leads to a small increase in residual stress at the outside diameter (OD), thus increasing R ratio at that location and reducing fatigue lifetime for crack growth originating at the OD.

Autofrettage should be conducted on the component as near to its final dimensions as is practicable in order to maximise bore hoop compressive stresses in the cases where reversed yielding may occur, generally  $c/a > 1.3$ , which encompasses virtually all autofrettaged gun tube geometries.

## FOOTNOTE

After preparation and presentation of this paper at this Symposium the new ASME pressure vessel code became available. Section KD-522.2 of this new code relates to Bauschinger effect corrections for autofrettaged tubes. The authors understand that the code specifications are closely related to the work referred to in references [2, 5 and 9] of this paper. It is clear that there are some significant differences between the new code and the predictions of this paper. The new code is directly comparable with several aspects of this paper, and these comparisons are presented in detail in reference [17]. In summary these comparisons indicate that the method proposed within this paper yields a good lower bound (conservative) fit to available experimental data whereas the new code provides a non-conservative upper bound. .

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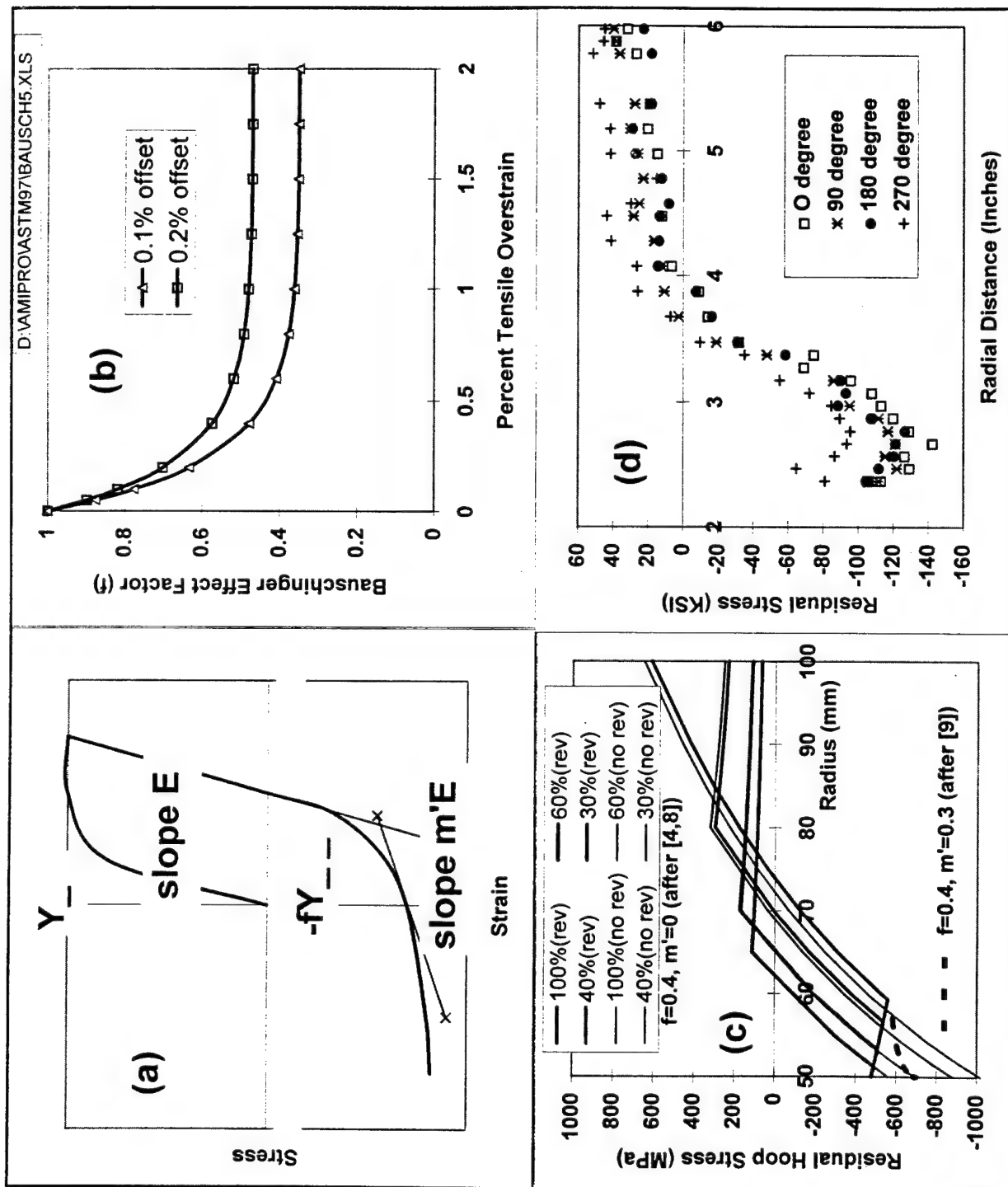


Figure 1. Bauschinger Effect: (a) Stress-Strain Curve for Typical Gun Steel; (b) Variation of BEF with Percentage Plastic Strain; (c) Form of Residual Hoop Stress Predictions; (d) Form of Residual Hoop Stress Measurements

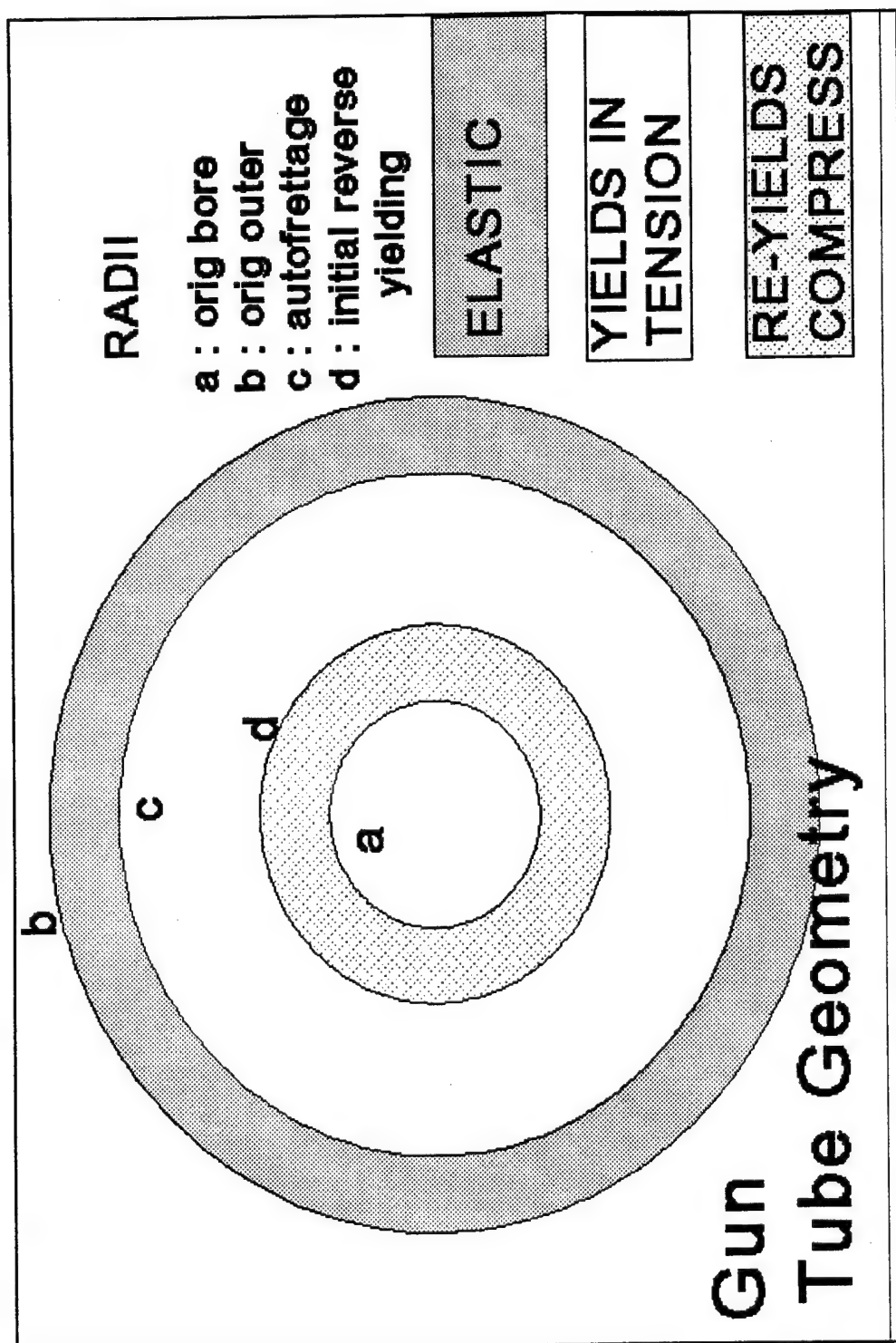


Figure 2. Tube Geometry

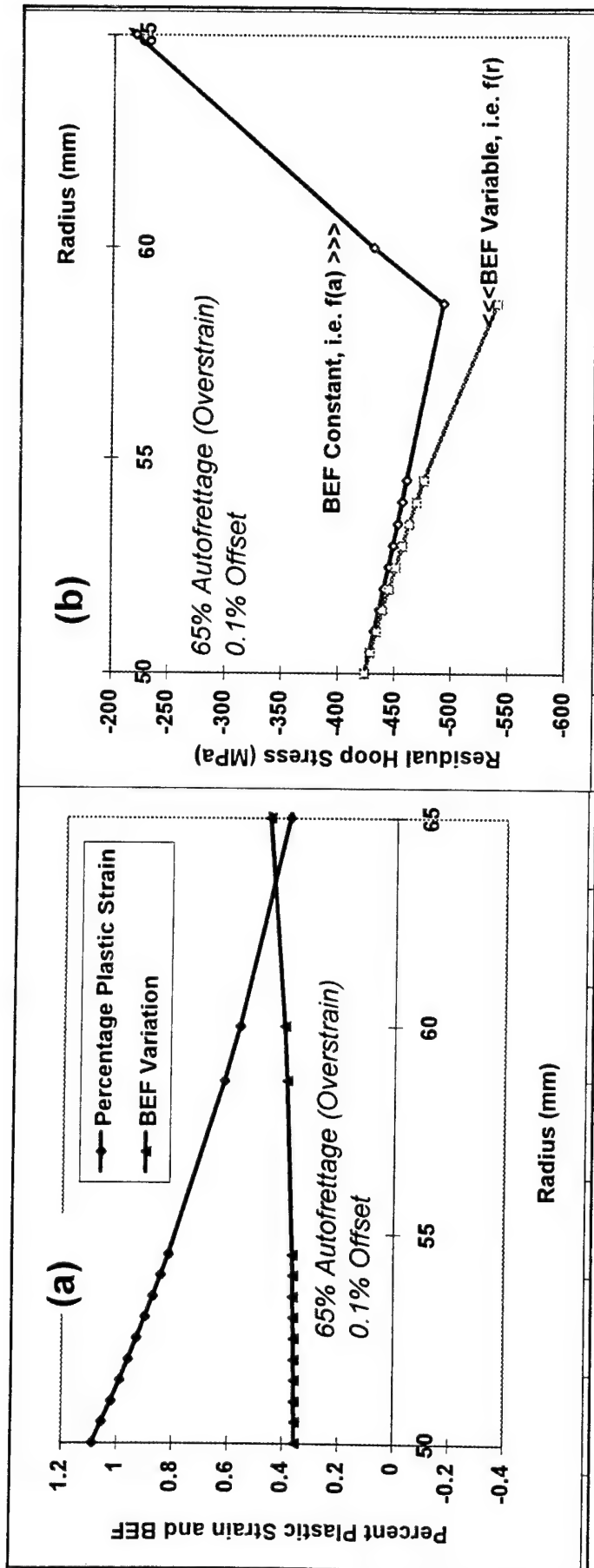


Figure 3. Plastic Strain, Bauschinger Effect Factor, and Residual Hoop Stress as a Function of Radius

# Autofrettage Stresses Incorporating Bauschinger

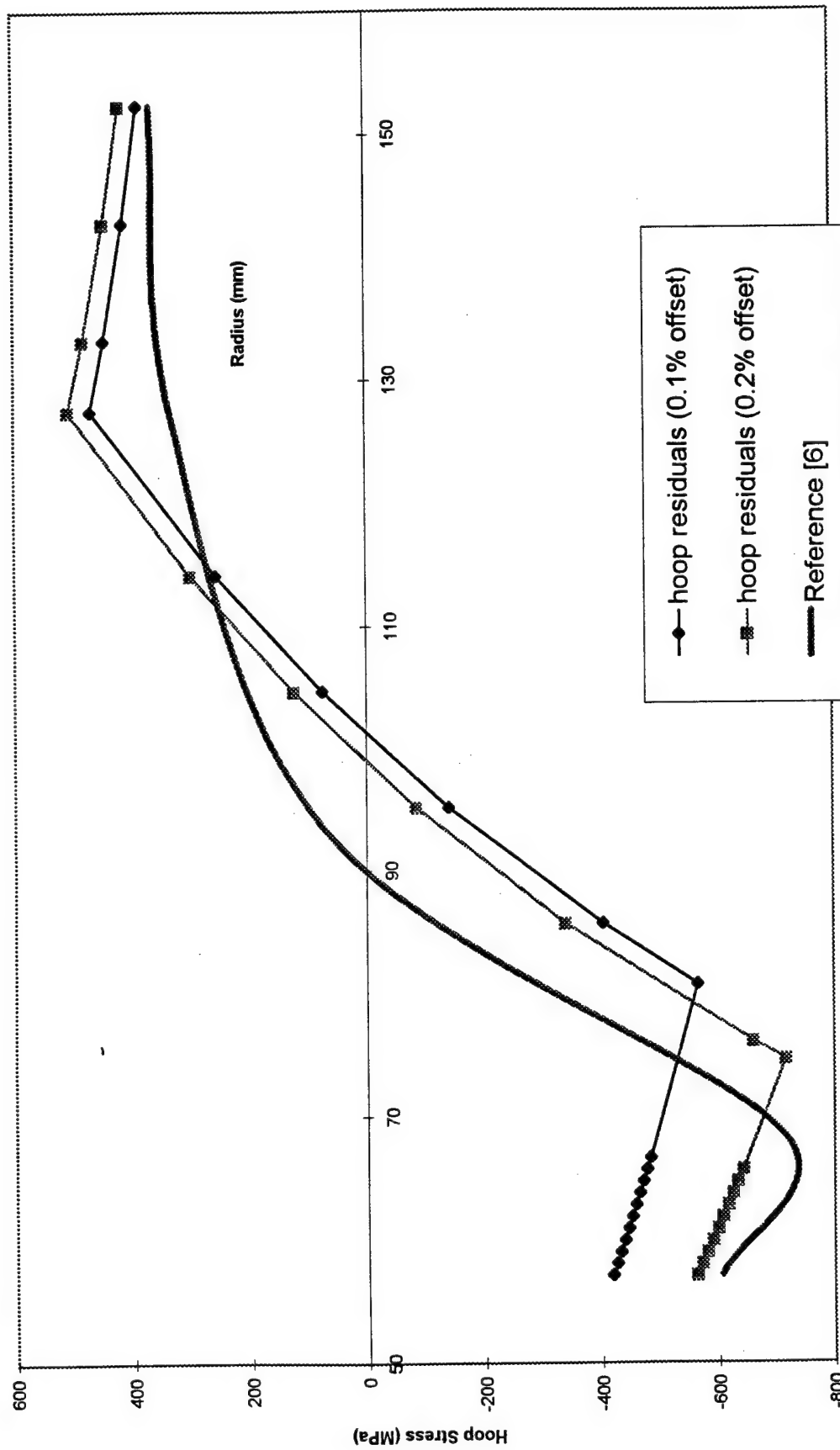
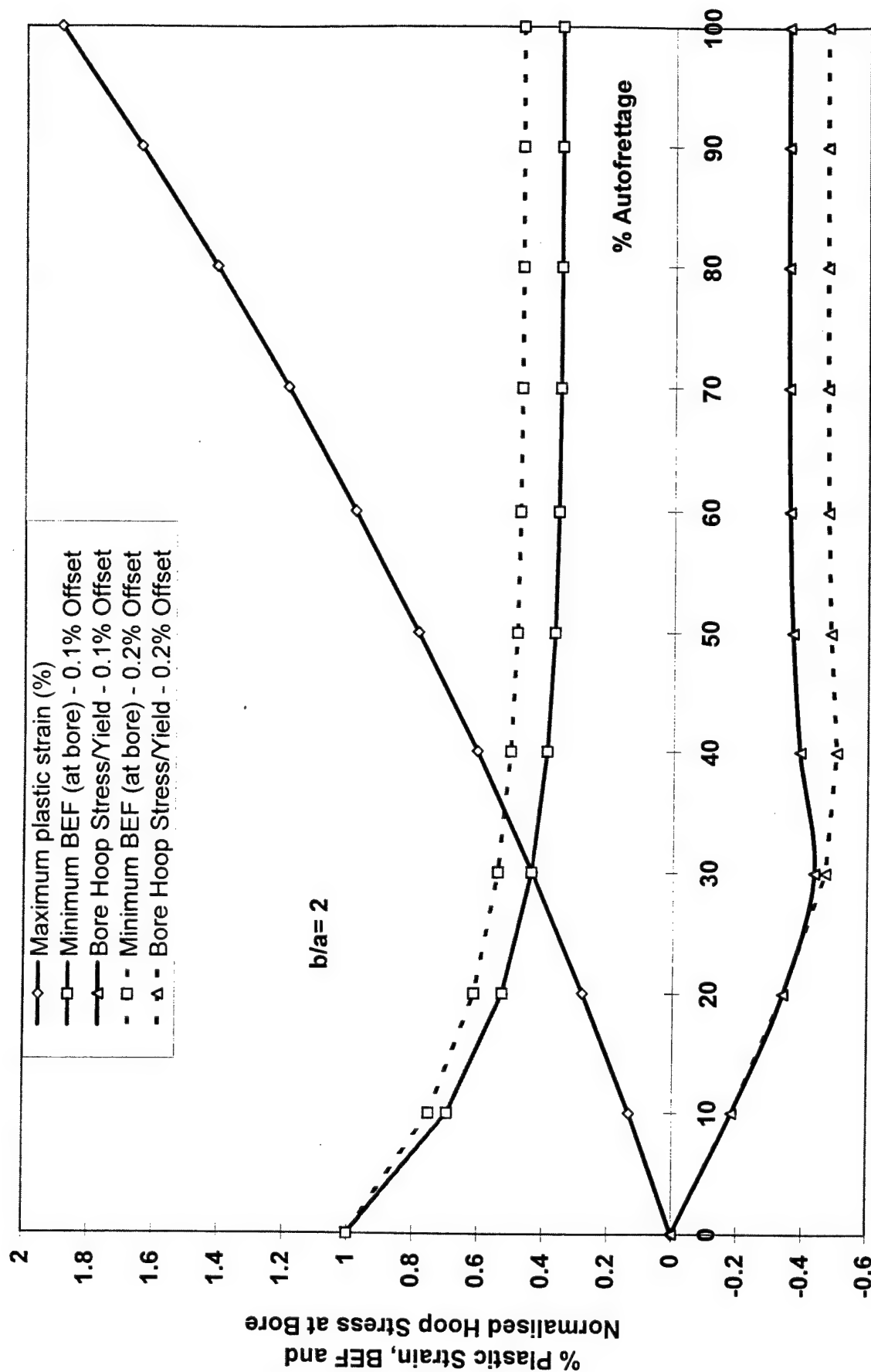


Figure 4. Comparison of Predicted Stresses with Experimental Data



BAUSCH5Y.XLS

Figure 5. Plastic Strain, Bauschinger Effect Factor, and Residual Hoop Stress at the Bore

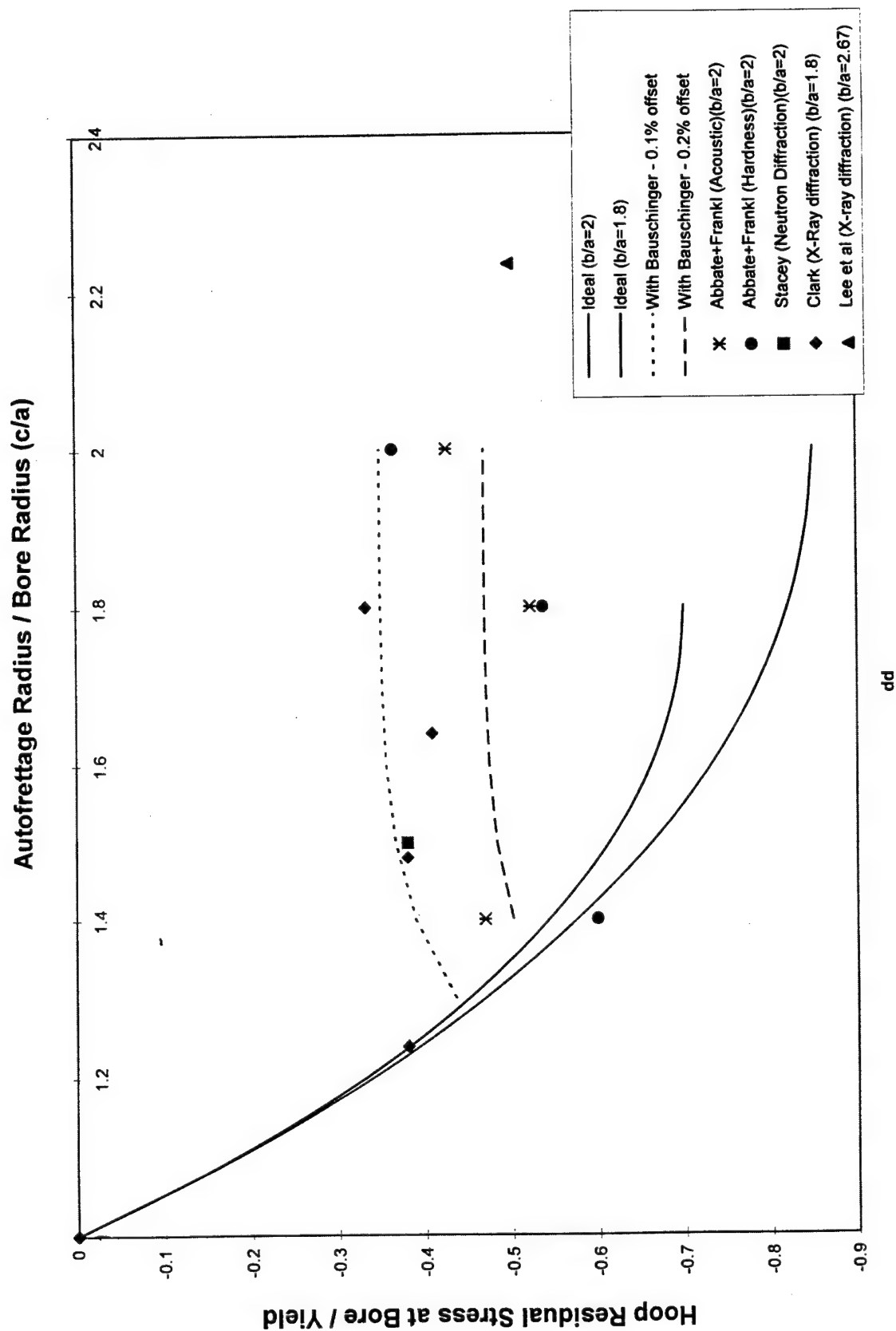


Figure 6. Residual Stress at the Bore - Bauschinger Model and Experimental Evidence

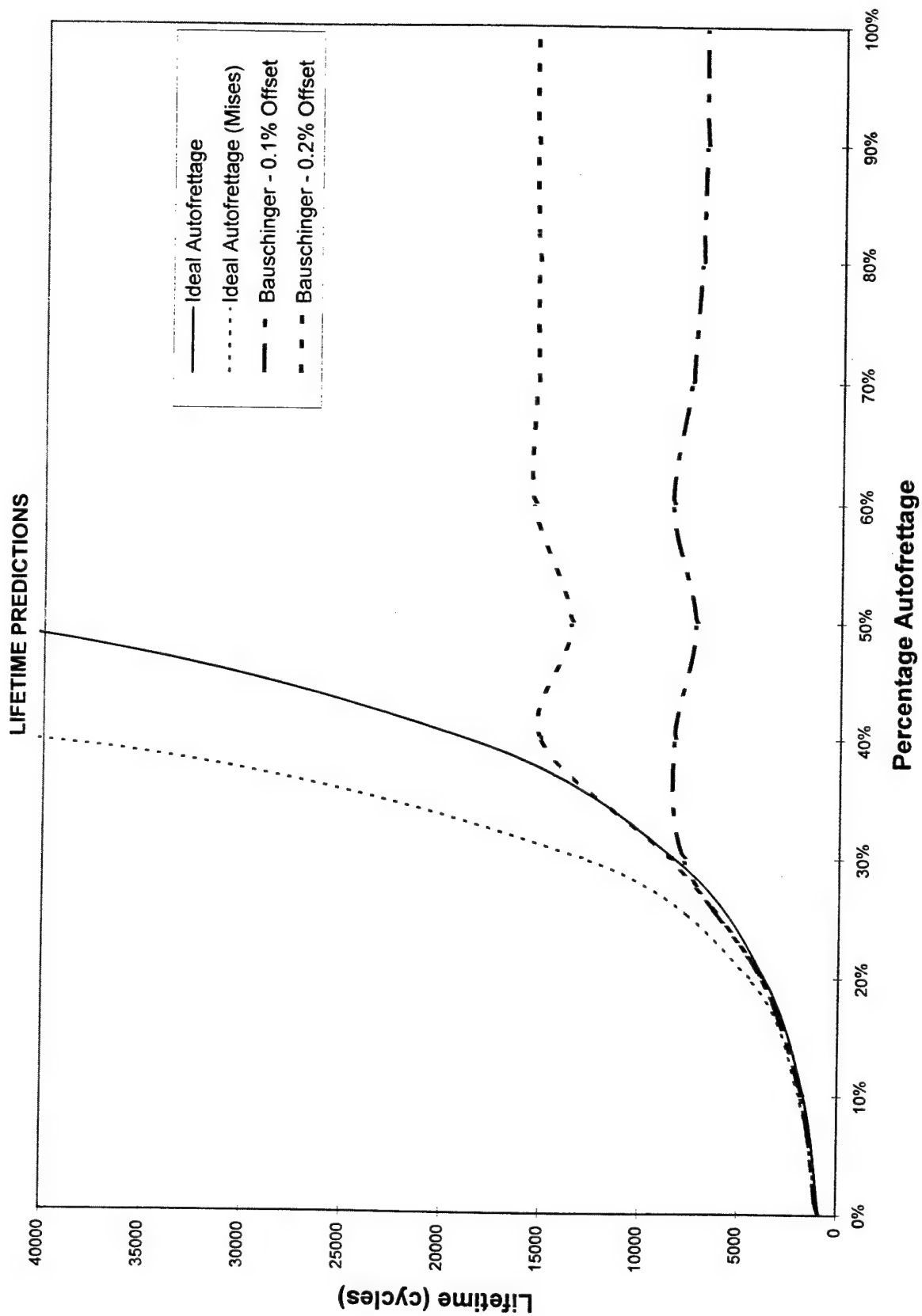


Figure 7. Predicted Lifetimes as a Function of Percentage Overstrain

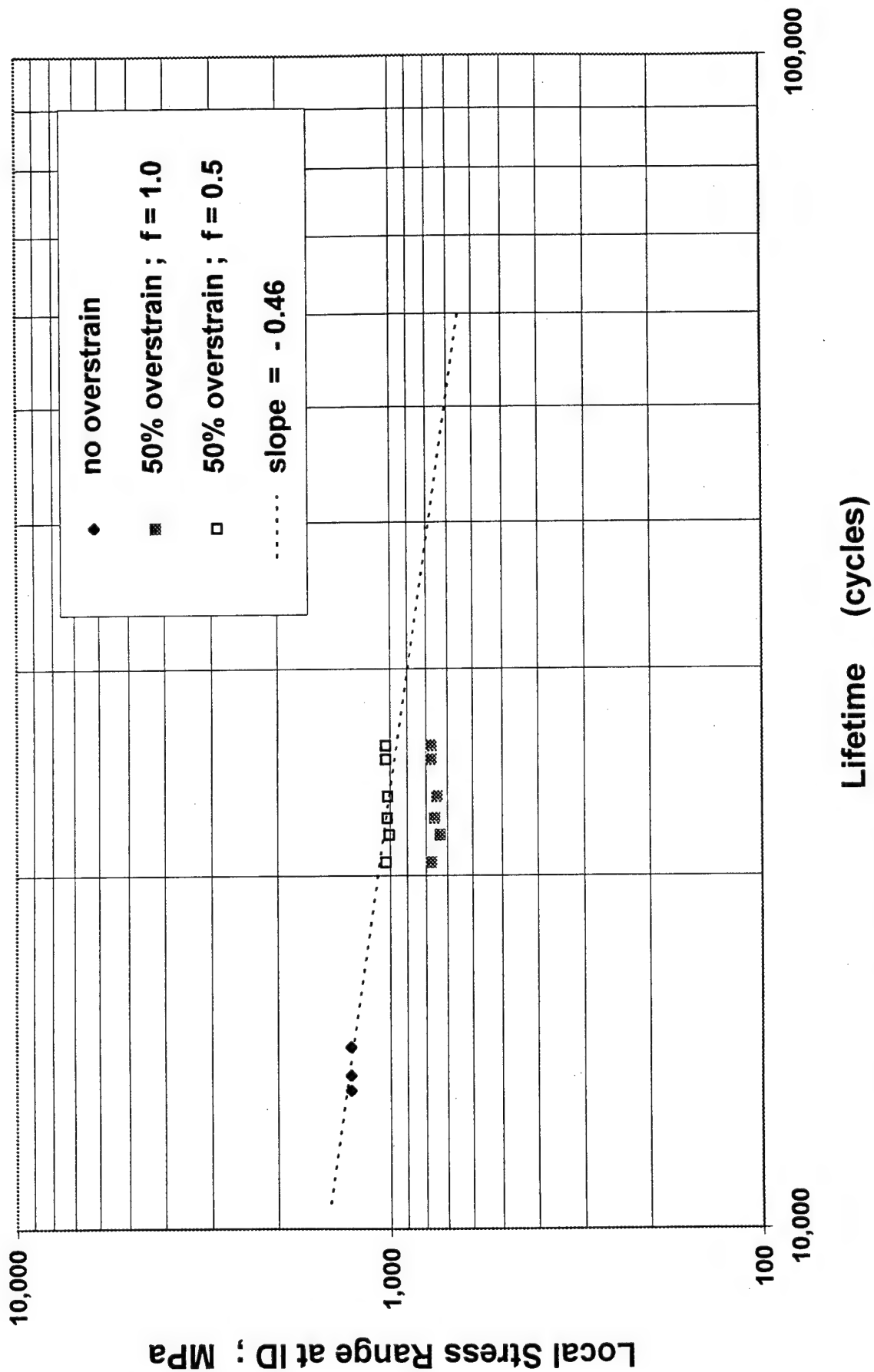


Figure 8. Effect of Overstrain and BEF on Measured Fatigue Lifetime of Pressurized Cylinders

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